New Advances in Uncertainty Analysis and Estimation

Overview:

Both sensor observation data and mathematical models are used to assist in the understanding of physical dynamic systems. However, observational data is often limited in terms of the kind and frequency of observations that can be taken and may only provide access to limited aspects of the system states. Also, any mathematical model used to represent the system dynamics is a reflection of numerous assumptions and simplifications to permit determination of a tractable model. These factors cause overall accuracy to degrade as the model states evolve. The fusion of observational data with state models promises to provide greater understanding of physical phenomenon than either approach alone can achieve. The most critical challenge here is to provide a quantitative assessment of how closely our estimates reflect reality in the presence of model uncertainty, discretization errors as well as measurement errors and uncertainty. The quantitative understanding of uncertainty is essential when predictions are to be used to inform policy making or mitigation solutions where significant resources are at stake.

This workshop will focus on recent development of mathematical and algorithmic fundamentals for uncertainty propagation, forecasting, and model-data fusion for nonlinear systems. The emphasis of this workshop will be on an intuitive understanding of the stochastic processes and practical applications of theory of stochastic processes in estimation and control area. The objectives are to develop a fundamental understanding of stochastic processes and its applications in the area of filtering and control of dynamical systems, to develop an appreciation for the strengths and limitations of state-of-the-art numerical techniques for uncertainty propagation and nonlinear filtering, to reinforce knowledge in stochastic systems with particular emphasis on nonlinear and dynamic problems, and to learn to utilize stochastic system analysis methods as research tools. After the completion of this workshop, audience should be able to apply the discussed methods to real engineering problems with the awareness of potential difficulties that might arise in practice. This workshop would cover topics from basic linear and nonlinear stochastic processes to well-known Kalman filtering methods to recently developed nonlinear estimation methods at a level of detail compatible with the design and implementation of modern control and estimation of dynamical systems. These diverse topics will be covered in an integrated fashion, using a framework derived from stochastic processes, estimation, control, and approximation theory. The reliability and limitations of various methods discussed will be assessed by considering various academic and engineering problems. At the end of this workshop, audience will be able to:

1. Understand and use the concept of stochastic processes to model engineering systems.
2. Learn to apply linear uncertainty propagation and filtering techniques to engineering problems.
3. Understand and derive numerical solution techniques to solve nonlinear uncertainty propagation and filtering problems.
4. Get exposed to implementation issues such as computational complexity, non-Gaussian uncertainty, reduction of filter dimension, colored noise, discretization etc

The numerical methods to solve the Kolmogorov equation, Perron-Frobenius operator, generalized Polynomial chaos and stochastic collocation methods will be discussed to determine evolution of state pdf due to probabilistic uncertainty in initial or boundary conditions, model parameters and forcing function. Recent advances in sampling methods like Conjugate Unscented Transformation (CUT) will be presented to compute multi-dimensional expectation integrals. A Bayesian framework is used to assimilate the noisy observation data from various sources with uncertain model forecasts to reduce the uncertainty associated with model-state estimates. By accurately characterizing the uncertainty associated with both process and measurement models, this workshop offers systematic design of low-complexity model-data fusion or filtering algorithms with significant improvement in nominal performance and computational effort. Various academic and engineering problems where traditional methods either fail or perform very poorly, will be considered to demonstrate the reliability and limitations of the newly established methods.

Format:

This workshop will consist of two half-day sessions. Focus of the first session will be to introduce the basic principles of stochastic systems, utility of Kolmogorov equation along with numerical methods to solve it and the basics of stochastic collocation. The second session will primarily focus on quadrature schemes to aid stochastic collocation, the basics of nonlinear filtering and application of various methodologies to real engineering applications. Speakers will lead sessions by giving short, concise presentations on the latest developments, followed by a forum of speakers-audience interactions on questions and concerns from participants, submitted either on-line or in person. Practical problems and examples will be used throughout the discussions to help participants connect novel principles to their own experiences.

Target Audience:

Practitioners looking for advanced methods to propagate non-Gaussian state and parameter uncertainties through nonlinear dynamical systems, researchers looking for new ideas and the connection between theory and practice; students looking for better understanding of the foundation of uncertainty quantification and the essence of nonlinear estimation engineering practice.

Tentative Schedule and Speakers:

Speakers: Dr. Puneet Singla (SUNY Buffalo), Dr. Raktim Bhattacharya (Texas A&M University)

SESSION 1. (8:00 – 12:00)

1) Introduction and Opening Remarks (8:00 – 8:10)
2) Uncertainty Modeling (8:10 - 8:30)
3) Short Review of Probability and Stochastic Processes (8:30 - 9:00)
4) Propagation of PDF by solving Kolmogorov Equation (9:00 – 10:30)
5) Spectral Representation and Moment Propagation (10:30 – 12:00)
6) LUNCH BREAK (On your Own 12:00 – 1:00)

SESSION 2. (1:00 – 5:00)

7) Quadrature methods (1:00 – 2:30)
8) Applications to Estimation & Filtering (2:30 – 4:00)
9) Discussions (4:00 – 5:00)

Abstracts for Talks:

Uncertainty Modeling: (Speaker: Raktim Bhattacharya) – There are many aspects of uncertainty in engineering systems that are broadly classified as aleatory or epistemic. Aleatory uncertainty includes uncertainty in initial conditions and parameters, which models structured uncertainty or known unknowns. Examples of epistemic uncertainty include unmodeled dynamics or incomplete knowledge of physical systems, or a source of uncertainty that has no known structure. We classify them as unknown unknowns. We will highlight these classes of uncertainties with examples from various engineering disciplines and discuss how they can be modeled for purposes of uncertainty quantification.

Short Review of Probability and Stochastic Processes (Speaker: Puneet Singla) – We will review basic concepts related to random variables, random process, conditional probability density function, Bayesian inference, entropy, Kullback-Fisher information, continuous and discrete random process, Brownian motion, and white noise. These will provide the necessary mathematical background for the material presented later in the workshop.

Propagation of PDF by solving Kolmogorov Equation (Speaker: Puneet Singla, Raktim Bhattacharya) – This talk will focus on recent development in computational methods for uncertainty characterization and forecasting for nonlinear systems. The central idea is to replace evolution of initial conditions for a large dynamical system by evolution of probability density functions (pdf) for state variables. The use of Fokker-Planck-Kolmogorov equation (FPKE) and Chapman-Kolmogorov equation (CKE) to determine evolution of state pdf due to probabilistic uncertainty in initial or boundary conditions, model parameters and forcing function will be discussed. Analytical solutions for the FPKE/CKE exist only for a stationary pdf and are restricted to a limited class of dynamical systems. Traditional numerical approaches based upon variational formulation which discretize the space in which the pdf lies, suffer from the "curse of dimensionality." In this talk, we will discuss that how one can make use of recent advances in approximation theory to not only break the "curse of dimensionality" but can also pose the the pdf evolution problem as a convex optimization problem with guaranteed convergence. In particular, the audience will be introduced to use of Gaussian mixture model based approaches to solve both the differential and integral form of the Kolmogorov equation.

For systems with weak diffusion, the FPKE can be simplified to the continuity equation. This results in considerable simplification in the solution of the governing equation. The continuity equation being first order linear can be solved using method of characteristics and Rothe's method. Additionally, we will describe use of maximum entropy basis functions to approximate the PDF evolution in a mesh less computational framework.

Spectral Representation and Moment Propagation (Speaker: Raktim Bhattacharya) – We will discuss approximation of random processes using KL and polynomial chaos expansions, and describe when these approximation techniques can be applied. We will discuss how these techniques can be applied to both linear and nonlinear systems. We will also highlight the computational complexity associated with polynomial chaos approximation, in particular with Galerkin projections, and make a case for stochastic collocation techniques. Examples illustrating strengths and weakness of spectral approximations will also be presented. Implementation details and accuracy of approximations will be discussed using few numerical examples.

Quadrature methods (Speaker: Puneet Singla) – This talk will introduce the theory of Gauss quadrature methods to evaluate expectation integral involving a generic density function. Quadrature methods involve an approximation of the expectation integral as a weighted sum of integrand values at specified points within the domain of integration. A quadrature rule is said to be exact to degree d, if it can only integrate all polynomials with degree $\leq$ d. For 1-D (1-Dimensional) integrals, one needs N quadrature points according to the Gaussian quadrature scheme to exactly reproduce the expectation integrals of polynomials with degree 2N-1 or less, . However, in generic n-D, one needs to take the tensor product of 1-D quadrature points and hence would yield a total of N^n quadrature points. This is a non-trivial number of points that might make the calculation of the integral computationally expensive, especially when the evaluation of function at each cubature point itself can be an expensive procedure. This talk will introduce recently developed Conjugate Unscented Transformation (CUT) approach to accurately evaluate expectation integrals in high dimension space while minimizing the number of simulations. Rather than using tensor products as in Gauss quadrature, the CUT approach judiciously selects specific structures to extract symmetric quadrature points. Several benchmark problems will be considered to highlight relative merits of various algorithms and their use in stochastic collocation.

Applications to Estimation & Filtering (Speakers: Puneet Singla and Raktim Bhattacharya) – This talk will introduce the concept of model-data fusion, which has its birth with the development of Kalman filter for linear system. We will discuss that how various uncertainty propagation methods introduced in prior sections can be used along with Bayes' rule to find system state and parameter estimates. In addition, the concept of maximum likelihood estimation and best linear unbiased estimator will be discussed. Relative merits of different approaches will be discussed while considering various benchmark problems.

Bio-sketches for Speakers:

Puneet Singla: Dr. Puneet Singla is an Associate Professor of Mechanical & Aerospace engineering at the University at Buffalo (UB), the State University of New York. He received his bachelor's degree in Aerospace Engineering from Indian Institute of Technology, Kanpur, India in 2000 and earned his master's and doctoral degree in Aerospace Engineering from Texas A&M University, College Station in 2002 and 2006, respectively. His research work includes three thrusts: 1) characterization and propagation of uncertainties through dynamical systems, 2) design of computationally efficient data assimilation algorithms for large scale problems, and 3) design robust methodologies for optimal sensor management while taking into account the uncertainties in the system dynamics. The research layer surrounding the focus areas includes approximation theory, study of stochastic systems, nonlinear filtering and control. The theoretical developments form the framework for diverse problems such as dispersion & transport of toxic material clouds through the atmosphere, tracking resident space objects, tumor motion modeling for conformal radiation therapy, flow control and control of robotic systems.

During his tenure at UB, he has secured several research grants as a PI or co-PI from the National Science Foundation (NSF), Air Force Office of Scientific Research (AFOSR), Air Force Research Laboratory (AFRL) and the National Geospatial Intelligence Agency (NGA). He is a recipient of the competitive NSF CAREER award for his work on Uncertainty Propagation and Data Assimilation for Toxic Cloud Prediction and the AFOSR Young Investigator Award for his work on Information Collection and Fusion for Space Situational Awareness. He has also been awarded the UB's "Exceptional Scholar" Young Investigator Award in recognition of his scholarly activities. He has authored over 100 papers to-date including 25 journal articles covering a wide array of problems, including: attitude estimation, nonlinear estimation, dynamics and control, adaptive control, approximation theory, including novel methods for solving the Fokker-Planck-Kolmogorov equation (FPKE) for uncertainty propagation. He is the principal author of a new textbook entitled "Multi-Resolution Methods for Modeling and Control of Dynamical Systems," (300 pages) published in August 2008 by CRC Press (Boca Raton, FL). He has received the best paper awards at the 2006 AIAA/AAS Astrodynamics Specialists Conference and 2009 International Information Fusion Conference for his work on uncertainty propagation.

Raktim Bhattacharya received his M.S. and PhD in Aerospace Engineering from the University of Minnesota in 2000 and 2003 respectively. He was a postdoctoral researcher in Control & Dynamical Systems at Caltech from 2003 to 2004. He spent 2004-2005 at United Technologies Research Center, East Hartford, CT, as a research scientist in the Controls and Embedded Systems Group. He joined the Aerospace Engineering department at Texas A&M University on 2005, and is currently an associate professor. He has published several journal & conference papers and book chapters in the area of probabilistic robust control, nonlinear estimation, UQ in hypersonic flight problems, nonlinear trajectory generation, anytime control algorithms, and receding horizon control methodologies. NASA and NSF have funded his research.